Chapter I.

Principles of Statics

1–1. Introduction

Engineering mechanics may be defined as the science which considers the effects of forces on rigid bodies. The subject divides naturally into two parts: statics and dynamics. In statics we consider the effects and distribution of forces on rigid bodies which are and remain at rest. In dynamics we consider the motion of rigid bodies caused by the forces acting upon them.

ENGINEERING MECHANICS

- Statics
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- Dynamics
  - Kinematics
    - Translation
    - Rotation
    - Plane Motion
  - Kinetics
    - Translation
    - Rotation
    - Plane Motion

FIG. 1–1.—Outline of engineering mechanics.

A visual introduction to the subject of engineering mechanics is represented in Fig. 1–1. The subject has two main divisions: statics and dynamics. These are subdivided into two subbranches. In statics we consider first the various types of force systems, then their application to the various conditions shown. Not all phases of statics, however, are represented in the diagram, merely the more common elements. Dynamics is similarly divided: there are kinematics (which deals with the pure motion of rigid bodies) and kinetics (which relates the motion to the applied forces). Each of these subdivisions deals primarily with the rigid body motions of translation, rotation, and plane motion. These terms are discussed at length in Part II. For the present, we confine ourselves to statics.

1–2. Fundamental Concepts and Definitions

Rigid Body. A rigid body is defined as a definite amount of matter the parts of which are fixed in position relative to each other. Actually, solid bodies are never rigid; they deform under the action of applied forces. In many cases, this deformation is negligible compared to the size of the body

and the body may be assumed rigid. Bodies made of steel or cast iron, for example, are of this type. The study of *strength of materials*, however, is based on the deformation (however small) of such bodies.

Force. Force may be defined as that which changes, or tends to change, the state of motion of a body. This definition applies to the *external* effect of a force. The *internal* effect of a force is to produce stress and deformation in the body on which the force acts. External effects of forces are considered in *engineering mechanics*; internal effects, in *strength of materials.*

The *characteristics* of a force are (1) its magnitude, (2) the position of its line of action, and (3) the direction (or sense) in which the force acts along its line of action.

The *principle of transmissibility* of a force states that the *external* effect of a force on a body is the same for all points of application along its line of action; i.e., it is independent of the point of application. The *internal* effect of a force, however, is definitely dependent on its point of application.

The *unit* of force commonly used in the United States is the pound, or multiples of the pound such as the kip (1000 pounds) or ton (2000 pounds) Units such as the gram and kilogram are also used. In this book we shall use the "foot-pound-second" system of units; i.e., the common unit of length is taken as the foot, of force as the pound, and of time as the second. If other units happen to be specified in problems, it is generally desirable to convert them into the foot-pound-second system before solving for the answer.

1–3. Force Systems

A force system is any arrangement where two or more forces act on a body or on a group of related bodies. When the lines of action of all the forces in a force system lie in one plane, they are referred to as being *coplanar;* otherwise they are *non-coplanar*. The coplanar system is obviously simpler than a non-coplanar system since all the action lines of the forces lie in the same plane. We shall consider first a discussion of coplanar systems; it will then be a relatively simple step to the discussion of non-coplanar or space systems of forces.

The force systems are further classified according to their lines of action. Forces whose lines of action pass through a common point are called *concurrent;* those in which the lines of action are parallel are called *parallel force systems;* and those in which the lines of action neither are parallel nor intersect in a common point are known as *non-concurrent force systems.*

1–4. Axioms of Mechanics

The principles of mechanics are postulated upon several more or less self-evident facts which cannot be proved mathematically but can only be

demonstrated to be true. We shall call these facts the fundamental axioms of mechanics. The axioms are discussed at length in subsequent articles as they are used. At this time we shall merely collate them for reference and state them in the following form:

1. The parallelogram law: The resultant of two forces is the diagonal of the parallelogram formed on the vectors of these forces.
2. Two forces are in equilibrium only when equal in magnitude, opposite in direction, and collinear in action.
3. A set of forces in equilibrium may be added to any system of forces without changing the effect of the original system.
4. Action and reaction forces are equal but oppositely directed.

1–5. Introduction to Free-Body Diagrams

One of the most important concepts in mechanics is that of the free-body diagram. This concept is discussed in detail in Chapter III where we first really use it. It is introduced here to help the beginner distinguish between action and reaction forces. To do so, it is necessary to *isolate* the body being considered. A sketch of the isolated body which shows only the forces acting upon the body is defined as a *free-body diagram.* The forces acting on the free body are the action forces, also called the applied forces. The reaction forces are those exerted by the free body upon other bodies.

The free body may consist of an entire assembled structure or an isolated

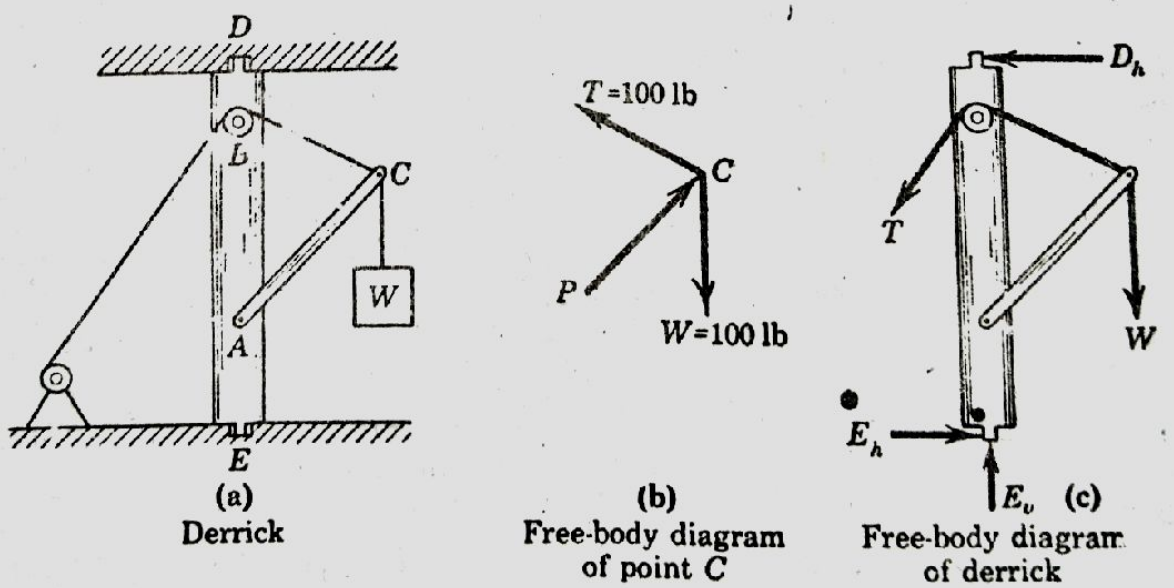

Fig. 1–2. — Free-body diagrams.

part of it. For example, consider the derrick shown in Fig. 1–2a. The free-body diagram of pin C (Fig. 1–2b) shows only the forces acting upon C These forces consist of the weight, the pull T exerted by the cable, and the force P exerted by the boom If the free-body diagram of the entire derrick

were desired, it would show only the forces acting on the derrick as in Fig. 1–2c.

1–6. Scalar and Vector Quantities

Scalars. Imagine two groups of marbles, one consisting of 10 marbles and the other of 5. If a common group is formed by mixing them, the resultant number will be 15 marbles, a result obtained by arithmetical addition. Quantities which possess magnitude only and can be added arithmetically are defined as scalar quantities.

Vectors. At point C of the derrick (Fig. 1–2b) suppose the weight W and the tension T were each 100 lb. What is the force P in the boom? By arithmetical addition the answer is 200 lb. This result, however, is incorrect, as can be determined by means of a measuring device placed in the boom. Actually the force in the boom would vary as the boom was lifted. The error is due to the fact that arithmetical addition was applied to quantities which possess *direction* as well as magnitude. Such quantities can be combined only by *geometric* addition, usually called vector addition. A vector of a quantity can be represented geometrically (i.e., graphically) by drawing a line acting in the direction of the quantity, the length of the line representing to some scale the magnitude of the quantity. An arrow is placed on the line, usually at the end, to denote the sense of the direction.

1–7. Parallelogram Law

The method of vector addition is based on what is known as the parallelogram law. The parallelogram law cannot be proved; it can only be demonstrated by experiment. It is one of the fundamental axioms of mechanics. One method of demonstrating the law is by means of the apparatus shown in Fig. 1–3. Tie three cords together and fasten the weights P, Q, and W to the free ends. (The sum of P and Q must be greater than W.) Place the cords to which P and Q are attached over the smooth pegs as shown and allow the system to reach a position of equilibrium.

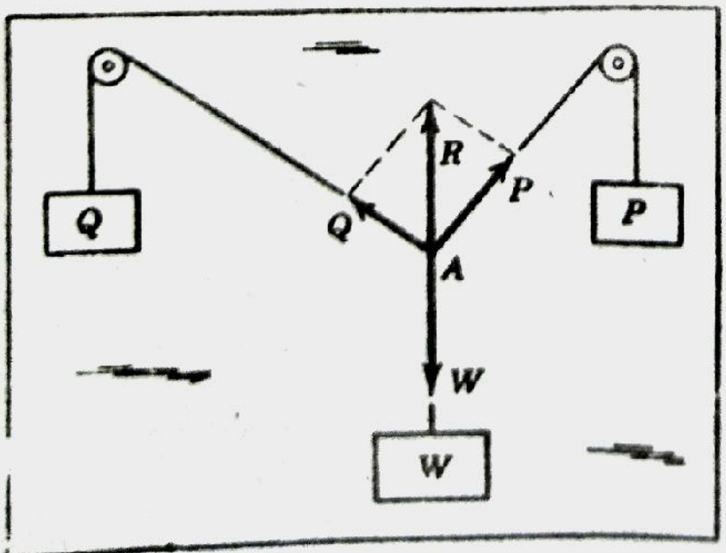

FIG. 1–3. — Parallelogram law.

The tensions in these cords will then be equal to the weights P and Q. Draw vectors P and Q to scale from point A where the cords are tied together and construct a parallelogram with these vectors as the initial sides. It will be found that the diago-

nal R of the parallelogram scales exactly to the value of W and is in line with the vector representing W.

From Axiom 2 which states that two equal, opposite, collinear forces are in equilibrium, we conclude that weight W will be perfectly supported by the force R. In other words, the net effect of the forces P and Q may be replaced by a single force R. Such a force is called a resultant. Therefore the resultant of two forces is the single force which will produce the same effect as the original forces.

The parallelogram law may now be stated as follows: *The resultant of two forces is the diagonal of the parallelogram formed on the vectors of these forces.*

1-8. Triangle Law

If we examine closely the parallelogram formed by forces P and Q as in Fig. 1-4b, we observe that side BC is parallel and equal to side AD. If the

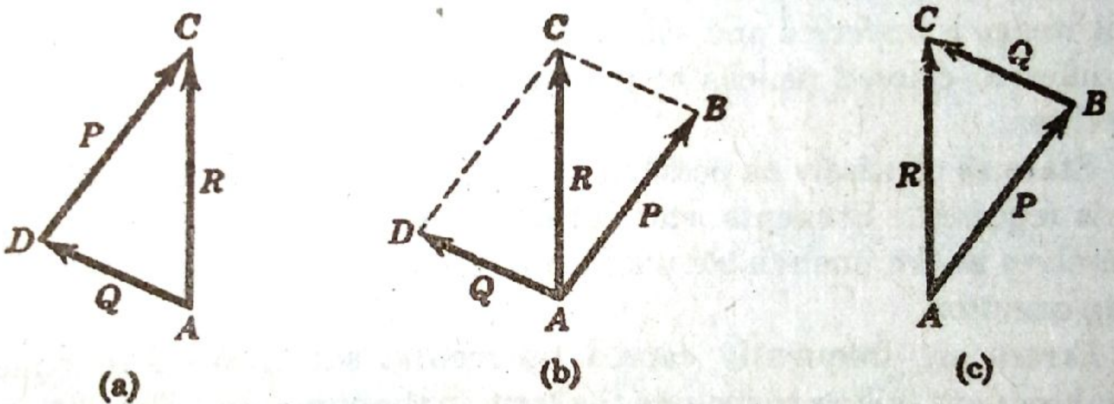

FIG. 1-4. — Triangle law.

triangle ABC were drawn alone as in Fig. 1-4c, the resultant R joining A to C would have the same magnitude and direction as the diagonal of the parallelogram $ABCD$. In this instance force Q has been represented by the free vector $\overline{BC}$. A *free vector* is defined as one which does not show the point of application of the vector, as distinguished from a *localized vector* which does.

It is also evident that, as $\overline{DC}$ is equal and parallel to P, the triangle ADC in Fig. 1-4a may also be used to determine R. In this case, P is taken as the free vector whereas Q is the localized vector.

We may now state the triangle law as a convenient corollary of the parallelogram law: If two forces are represented by their free vectors placed tip to tail, their resultant vector is the third side of the triangle, the direction of the resultant being from the tail of the first vector to the tip of the last vector.

Special Case. If the angle between two forces becomes zero or 180° the forces act along the same line; i.e., the forces are collinear. By takin

one direction as positive and the other direction as negative, it will be apparent that the resultant of two collinear forces is their algebraic sum. This application of the triangle law is extensively used in analytical solutions.

1–9. Solution of Problems

One of the first things a student should acquire is the ability to organize his work in a neat and orderly fashion. Properly arranged work not only helps to eliminate personal errors but also permits easy checking by another person — a frequent occurrence in engineering offices. To aid the student to achieve orderly work habits the following suggestions are offered:

1. After identifying the problem, start by constructing a neat diagram of the quantities involved. This diagram should be of sufficient size so that pertinent data and dimensions may be added without affecting its legibility. A freehand sketch is usually acceptable, although the use of a straightedge is preferred and will take little additional time. Some students use different-colored pencils to distinguish between known and unknown quantities.

2. State as concisely as possible what data are given and what information is required. Students who fail to realize what is required often find themselves in the unenviable position of obtaining the right answer to the wrong question.

3. Errors are frequently caused by mental substitution in equations and subsequent failure to include the term in the equation. For this reason, write out the equation you intend to use before substituting in it. This will also make the process apparent to any person who may check it. If an equation is not used, write a short note indicating the principle used or the operation performed. This short statement of theory — be it equation, principle, or operation — may be put at the left side of the sheet and the numerical work placed in line with it at the right of the page. In this book, whenever an equation or principle is used in the solution of a problem, it is stated at the left of the page in brackets and is followed by the solution in the same line. You cannot be urged too strongly to use this "theory-solution" technique when solving problems.

Experience has also shown that many students have difficulty in obtaining accurate numerical results even though they have correctly applied the principles. To indicate the way to more accurate computation, the illustrative problems discuss the technique of solution as well as the application of principles. The following articles offer several additional items for the student's guidance

1–10. Dimensional Checks

The equations used in engineering computations must be dimensionally homogeneous; that is, the units on each side must be of the same dimensional form. An easy way to check the dimensions in an equation is to substitute the dimensional equivalents of each term and then multiply or divide these equivalents as though they were algebraic quantities. This process determines the dimensional unit of each term.

For example, consider the equation $v^2 = v_o^2 + 2\,as$ where v and v_o are in feet per second, a in feet per second squared, and s in feet. The numeral 2 is a dimensionless number.[1] Substituting dimensionally in the equation, we get

$$\frac{\text{ft}^2}{\text{sec}^2} = \frac{\text{ft}^2}{\text{sec}^2} + \frac{\text{ft}}{\text{sec}^2} \times \text{ft}$$

which checks the equation since each term is in the same dimensional units.

A similar process may be used to determine the unit of an expression. For example, determine the dimensional unit of kinetic energy if it is expressed by the relation $\text{KE} = \frac{W}{2\,g} v^2$ where W is in pounds, v in feet per second, and g in feet per second squared. Substituting dimensionally, we have

$$\text{KE} = \frac{\text{lb}}{\text{ft per sec}^2} \times \frac{\text{ft}^2}{\text{sec}^2} = \frac{\text{lb}}{\text{ft}} \times \text{sec}^2 \times \frac{\text{ft}^2}{\text{sec}^2} = \text{ft-lb} \quad \textit{Ans.}$$

Even the definition of acceleration, $a = \frac{d^2s}{dt^2}$ (Art. 9–3), can be checked dimensionally. Here d^2s is a second differential of length, whereas dt^2 is a square of a differential of time. With units of length in feet and time in seconds, dimensional substitution in the definition for a yields

$$a = \frac{\text{ft}}{\text{sec}^2} \quad \textit{Ans.}$$

1–11. Conversion of Units

Occasionally it is necessary to convert a term from one system of units to another to make an equation dimensionally correct. The conversion is accomplished by multiplying the given term by unity where unity is a ratio of units (of the same kind but different in size) containing the required units and those given.

[1] In Art. 10–2, the numeral 2 is shown to be a factor of integration.

For example, convert a velocity of 60 miles per hour to units of feet per second. Begin by writing

$$v = 60\,\frac{\text{miles}}{\text{hour}}$$

To express v in feet per second, multiply the right side by the following ratios, each of which has the value of unity.

$$v = 60\,\frac{\text{miles}}{\text{hour}} \times \left(\frac{5280\text{ ft}}{\text{mile}}\right) \times \left(\frac{\text{hour}}{3600\text{ sec}}\right)$$

Canceling out like units, we obtain

$$v = 60 \times \frac{5280}{3600} = 88\,\frac{\text{ft}}{\text{sec}} \quad \textit{Ans.}$$

The ratio $\frac{5280}{3600}$ is the conversion factor by which miles per hour must be multiplied to yield feet per second. It is not necessary to remember conversion factors once the method is mastered.

As another example, consider the equation $\theta = \omega_o t + \frac{1}{2}\alpha t^2$ where ω_o is in revolutions per minute, t in seconds, and α in revolutions per minute per second. It is required to express θ in radians. Substituting dimensionally, we have

$$\theta\text{ (rad)} = \omega_o\left(\frac{\text{rev}}{\text{min}}\right) \times t\text{ (sec)} + \frac{1}{2} \times \alpha\left(\frac{\text{rev}}{\text{min} \times \text{sec}}\right) \times t^2\text{ (sec}^2\text{)}$$

Converting by multiplying by ratios having the value unity gives

$$\theta\text{ (rad)} = \omega_o\left(\frac{\text{rev}}{\text{min}} \times \frac{2\,\pi\text{ rad}}{\text{rev}} \times \frac{\text{min}}{60\text{ sec}}\right) \times t\text{ (sec)} +$$

$$\frac{1}{2} \times \alpha\left(\frac{\text{rev}}{\text{min} \times \text{sec}} \times \frac{2\,\pi\text{ rad}}{\text{rev}} \times \frac{\text{min}}{60\text{ sec}}\right) \times t^2\text{ (sec}^2\text{)}$$

$$\theta\text{ (rad)} = \omega_o t \times \frac{2\,\pi}{60}\text{ (rad)} + \frac{1}{2}\alpha t^2 \times \frac{2\,\pi}{60}\text{ (rad)} \quad \textit{Ans.}$$

Each term is now expressed in units of radians. In this example, θ was obtained in radians by multiplying each right-hand term by the factor $\frac{2\pi}{60}$.

1–12. Numerical Accuracy

Significant Figures. It frequently happens that numerical work is computed to a greater degree of accuracy than is warranted by the given data. The accuracy of the final result depends upon the least accurate figure used in the computation. For example, if the value of the gravita-

tional constant g is taken as 32.2, the answer should be computed to three significant figures only. No increase in accuracy is attained by carrying more figures through a computation than those in the least accurate data. If for some reason four figures are carried through a computation where the data in the example are accurate to only three figures, the answer should be rounded off to three figures.

If a result is found to be 24.2, for example, the number indicates that the result is greater than 24.15 but less than 24.25; the last 2 of the result is doubtful. On the other hand, 24.20 means that the result is greater than 24.195 but less than 24.205; obviously, the last zero should not be added unless it is a significant figure.

Location of the Decimal Point. In sliderule work difficulty is sometimes experienced in locating the decimal point. It is not advisable to rely upon systems that locate this point by counting the number of times the slide extends to the left or right, or by counting the significant figures, etc. A better method is to compute an approximate answer for the problem by rounding off the figures and canceling as far as possible. This method not only will locate the decimal point but will check the sliderule work.

For example, compute the value of

$$\frac{2.196 \times 48.2 \times 289}{3420 \times 68.2}$$

Rounding off figures gives approximately

$$\frac{2 \times 50 \times 300}{3500 \times 70} = \frac{6}{49} \approx \frac{6}{50} = 0.12$$

By the sliderule, the numerals in the result are 131; hence, to resemble the approximate value, the answer must be 0.131.

Solution of Quadratic Equations. Quadratic equations may be solved by formula, sliderule, completing the square, etc. Completing the square is probably as fast and accurate as any other method. It is obtained as follows:

Consider a quadratic equation of the form $x^2 + ax - b = 0$. Rearranging this, we get

$$x^2 + ax + \left(\frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + b$$

whence

$$\left(x + \frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + b$$

$$x + \frac{a}{2} = \pm\sqrt{\left(\frac{a}{2}\right)^2 + b}$$

from which the value of x is easily found.

For example, solve for the roots of

$$x^2 - 22.8\,x - 31.3 = 0$$

From the above discussion, we obtain

$$\begin{aligned} x^2 - 22.8\,x + (11.4)^2 &= (11.4)^2 + 31.3 \\ (x - 11.4)^2 &= 130 + 31.3 = 161.3 \\ x - 11.4 &= \pm 12.7 \\ x &= 24.1 \text{ and } x = -1.3 \quad \textit{Ans.} \end{aligned}$$

Trigonometric Functions. When a table of trigonometric functions is not available, it is useful to know how to compute the value of sin 86°, let us say, by means of a sliderule. This may be done as follows:

$$\begin{aligned} &\text{Since} & \sin 2\,\theta &= 2 \sin \theta \cos \theta \\ &\text{we have} & \sin 86° &= 2 \sin 43° \cos 43° \\ & & &= 2 \times 0.682 \times 0.731 \doteq 0.998 \quad \textit{Ans.} \end{aligned}$$

For angles less than 6° a similar technique is available. Since $\sin \theta \approx \theta$ for small angles (where θ is in radians), the value of sin $\frac{1}{2}$°, for example, is given by the equation

$$\frac{1°}{2} = \frac{1}{2}\,\text{deg} \times \frac{2\,\pi\ \text{rad}}{360\ \text{deg}} = \frac{\pi}{360}\ \text{rad}$$

Therefore,

$$\sin \frac{1°}{2} = \frac{\pi}{360} = 0.00873 \quad \textit{Ans.}$$

Likewise for cosine functions of small angles, we note that $\cos \theta \approx 1 - \dfrac{\theta^2}{2}$. Hence, in computing the value of cos $\frac{1}{2}$° we get

$$\begin{aligned} \cos \frac{1°}{2} &= 1 - \frac{1}{2}\left(\frac{\pi}{360}\right)^2 = 1 - 0.0000382 \\ &= 0.9999618 \quad \textit{Ans.} \end{aligned}$$

TRIGONOMETRIC FUNCTIONS

Angle	15°	30°	45°	60°	75°
sin	0.259	$\frac{1}{2} = 0.500$	$\frac{1}{2}\sqrt{2} = 0.707$	$\frac{1}{2}\sqrt{3} = 0.866$	0.966
cos	0.966	$\frac{1}{2}\sqrt{3} = 0.866$	$\frac{1}{2}\sqrt{2} = 0.707$	$\frac{1}{2} = 0.500$	0.259
tan	0.268	$\frac{1}{3}\sqrt{3} = 0.577$	1.000	$\sqrt{3} = 1.732$	3.732

$$\sin(-\theta) = -\sin\theta; \quad \cos(-\theta) = \cos\theta; \quad \tan(-\theta) = -\tan\theta$$

$$\sin^2\theta + \cos^2\theta = 1; \quad 1 + \tan^2\theta = \sec^2\theta; \quad 1 + \operatorname{ctn}^2\theta = \csc^2\theta$$

$$\sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta; \quad \cos^2\theta = \tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots$$

DIFFERENTIALS AND INTEGRALS

$$\frac{dx^n}{dx} = nx^{n-1}; \qquad \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}; \qquad \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{dx}{x} = \log_e x + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C \qquad \int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\int u\,dv = uv - \int v\,du$$